# The Disintegration of the Deuteron by Impact

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High energy deuterons can be disintegrated by their impact with nuclei. For deuteron energies $\sim 2\times 10^7$ e.v., the corresponding neutron yield can be of the order of 1 percent. The probability of the process can be calculated by taking advantage of the fact that the nuclear field varies little over the deuteron, and is then given quite simply in terms of the photoelectric absorption and the matrix elements of the nuclear field acting on the deuteron. For low energies the neutron yields are small, amounting to $3\times 10^{-9}$ for $3\times 10^6$ e.v. deuterons in Al, and to $10^{-6}$ for $4\times 10^6$ e.v. deuterons in carbon.

ACCORDING to the experiments of Chadwick and Goldhaber deuterons may be disintegrated by the gamma-rays of ThC''. The threshold for this photoelectric effect corresponds to the mass defect $I$ of the deuteron with respect to the proton and neutron into which it disintegrates, and lies roughly at $2\times 10^6$ e.v. Whenever deuterons of energy greater than this pass through matter, they may be disintegrated by their impact with atomic nuclei. It is this process which we wish to consider.

When the minimum time of impact $T$ is very short compared to the periods of the deuteron $\tau$, then we may determine the mean energy lost by the deuteron for its disintegration by applying the familiar method by which Bohr derived the atomic energy losses of fast particles. This gives for the energy loss per cm

$$\Delta E/\Delta x = [4\pi Z^2 e^4/(2M)v^2] N \ln (\tau/T). \quad (1)^*$$

Here $N$ is the number of nuclei/cm³, $Ze$ their charge, $v$ the deuteron velocity and $2M$ the deuteron mass. Further

$$T = Ze^2/Mv^3, \quad \tau = \hbar/Ik, \quad (2)$$

where $k$ is a constant of order unity, which depends on the structure of the deuteron. The cross section for disintegration is thus in this limit

$$\sigma = (2\pi Z^2 e^4/Mv^2 k_1 I)\lambda\,; \quad \lambda = \ln (Mv^3\hbar/Ze^2 kI) \quad (3)$$

where $k_1$ is again of order unity, and $k_1 I$ is the mean energy which the deuteron absorbs on disintegration. If we write[1] for the energy loss to atomic electrons

$$\frac{\Delta' E}{\Delta x} = \frac{4\pi N Z e^4}{mv^2}\Lambda\,; \quad \Lambda = \ln \frac{mv^3}{Ze^2 \kappa R}\,; \quad \kappa \sim 5.3\,; \quad R = \frac{me^4}{4\pi\hbar^3} \quad (4)$$

then the yield of neutrons from a thick target, for an initial deuteron energy $W$, is

$$Z(m/2M)(W/k_1 I)L \quad (L\sim 1) \quad (5)$$

where $L$ is the average over the energy range of the deuteron of the ratio $\lambda/\Lambda$. For high energies ($\sim 3\times 10^7$ e.v.) and large $Z$ this gives yields of the order of 1 percent. It therefore seems of interest to investigate this process for lower energies, where the assumption $T \ll \tau$ can no longer be made.

We can make this calculation very simply insofar as we may neglect the variation of the field of the nucleus over the deuteron. This in turn will be permissible whenever the dimensions $d$ of the deuteron are small compared to the distance of closest approach with the nucleus. Since $d$ cannot be smaller than $\hbar/(MI)^{1/2}$, we must have

$$\hbar/(MI)^{1/2} \ll Ze^2/Mv^2. \quad (6)$$

This condition does not contradict $T \ll \tau$ when

$$n = Ze^2/\hbar v \gg 1. \quad (7)$$

The condition (7) is well satisfied except for the lightest elements, for all deuteron energies which are likely to be available; and we shall assume the validity of (7) in the following work.

* This is the mean energy transferred, by the impact of a nucleus of velocity $v$ on a deuteron initially at rest, to the relative motion of neutron and proton with respect to their center of mass; the mean work done on the proton is just twice this.

[1] F. Bloch, Zeits. f. Physik **81**, 363 (1933).

Under these circumstances we can compute the field acting on the deuteron as though it were a point charge moving in the field of the nucleus; let us call the matrix component of this field which corresponds to an energy loss of the deuteron $=h\nu$, $\mathcal{E}_\nu$. This field will disintegrate the deuteron; if $\sigma_\nu$ be the cross section for photo-effect, the cross section for impact disintegration will be

$$\sigma=(c/4\pi h)\int_{I/h}^{W/h}\sigma_\nu|\mathcal{E}_\nu|^2 d\nu/\nu. \tag{8}$$

Now Bethe and Peierls have given[2] a theoretical calculation of $\sigma_\nu$, which is based on the assumption that the forces between proton and neutron act appreciably only over a region small compared to the size of the deuteron and the wavelength of the photoneutron: they find

$$\sigma_\nu=\frac{2h^2\alpha}{3\pi MI}\frac{x^{\frac{3}{2}}}{(1+x)^3};\quad x>0;\quad x=h\nu/I-1. \tag{9}$$

Since when we insert this in (8) the integrand falls off fairly rapidly with $\nu$, we may use (9) even for frequencies higher than those for which its derivation is valid; for the true value of $\sigma_\nu$ will give an integrand in (8) which falls off still more rapidly.

The matrix elements $\mathcal{E}_\nu$ are just $2M/e$ times the matrix elements of the acceleration of the deuteron; these are known[3] from the theory of the continuous x-ray spectrum, and assume a relatively simple form when (7) is valid. We have to remember, however, that the deuteron and nucleus have like charges, whereas the calculations on the continuous x-spectrum are made for two charges which attract each other. For our case we find

$$|\mathcal{E}_\nu|_2=\frac{16\pi^2}{3^{\frac{1}{2}}}\frac{z^2e^2}{v^2}\exp\left\{-2\pi n\times\left[\left(1-\frac{(1+x)I}{W}\right)^{-\frac{1}{2}}-1\right]\right\}g'\left(\frac{nI(1+x)}{2W}\right). \tag{10}$$

The function $g'(y)$ appears in the classical theory of the x-ray spectrum:

$$g'(y)=\tfrac{1}{4}\pi(3)^{\frac{1}{2}}iyH_{iy}{}^{(1)}(iy)H_{iy}{}^{(1)\prime}(iy). \tag{11}$$

A rough plot of $g'$ has been given[4] by Kramers: for large $y$, $g'=1$; and $g'$ does not deviate sensibly from this value except for very small values of $y$, where it is given by

$$g'(y)\rightarrow(3^{\frac{1}{2}}/\pi)\ln 2/yC,\quad \ln C=0.577. \tag{11a}$$

From (8), (9) and (10) we have

$$\begin{aligned}\sigma&=(16\pi Z^2e^4/3(3)^{\frac{1}{2}}Mv^2I)\varphi(Z,W),\\ \varphi(Z,W)&=\int_0^{W/I-1}\frac{x^{\frac{3}{2}}}{(1+x)^4}dx\\ &\quad\exp 2\pi n\{1-[1-(I/W)(1+x)]^{-\frac{1}{2}}\}\\ &\quad\times g'(nI/2W)(1+x)).\end{aligned} \tag{12}$$

The integral for $\varphi(Z, W)$ may be evaluated for the limiting cases $\frac{nI}{2W}\gg\ll 1$, but except in these cases can hardly be written in closed form.

For $\pi nI/W\gg 1$ we find

$$\sigma=\frac{4}{\pi(3)^{\frac{1}{2}}}\frac{Z^2e^4}{Mv^2I}\left(\frac{nI}{W}\right)^{-5/2}\exp 2\pi n[1-(1-I/W)^{-\frac{1}{2}}]. \tag{13}$$

For $nI/2W\ll 1$, on the other hand

$$\begin{aligned}\sigma&=\frac{\pi}{3}\frac{Z^2e^4}{Mv^2I}\ln\frac{W}{Ink}\qquad k=e^{-1/6}C\sim 1.51\\ &=\frac{\pi}{3}\frac{Z^2e^4}{Mv^2I}\ln\frac{Mv^3\hbar}{Ze^2kI}.\end{aligned} \tag{14}$$

This is in fact of the form (3), with $k=1.51$, $k_1=6$.

If we take $Z=13$, $I=2\times10^6$, $W=3\times10^6$, we may apply (13). This gives $\sigma\sim\frac{3}{4}\times10^{-27}$ cm$^2$, and with (4) gives a neutron yield of about $3\times10^{-9}$. For $Z=6$, $W=4\times10^6$, (13) is inapplicable. Here (12) and (4) give a yield of about $10^{-6}$. The increase in yield with energy is very rapid.

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[2] H. Bethe and R. Peierls, Proc. Roy. Soc. **A148**, 146 (1935).

[3] H. A. Kramers, Phil. Mag. **46**, 836 (1923) and G. Wentzel, Zeits. f. Physik **27**, 257 (1924) give the classical theory. Gaunt gives the quantum theoretic treatment, for unlike charges, for $n\gg1$: J. A. Gaunt, Zeits. f. Physik **59**, 508 (1930). For $h\rightarrow 0$ the exponent in (10) goes to $2\pi^2(Ze^2/Mv^3)\nu$.

[4] Kramers, reference 3, pp. 858–860.